Effect of Nuclear Elastic Scattering on Ion Heating in Thermonuclear Plasmas

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Abstract—Distortion in the fuel-ion (deuteron and triton) velocity distribution functions due to nuclear elastic scattering (NES) by beam ion and α-particle during neutral-beam-injection (NBI) plasma heating operation is studied by simultaneously solving the Boltzmann-Fokker-Planck (BFP) equations for deuteron, triton, α-particle and beam ion in the deuterium-tritium (DT) thermonuclear plasmas. Using the obtained distribution functions, enhancement in the T(d,n)⁴He fusion reaction rate coefficient between background deuteron and triton is examined. It is shown that the enhancement in the T(d,n)⁴He reaction rate coefficient becomes appreciable in low-density operations, i.e. nₑ ≤ 5x10¹⁹m⁻³.

Keywords - nuclear elastic scattering (NES); neutral-beam injection (NBI); deuterium-tritium (DT) thermonuclear plasma

I. INTRODUCTION

An intense neutral beam injected into thermonuclear plasma plays an important role in various stages of fusion reactor operations. The beam particles slow down, deposit most of their energy via Coulombic collisions, and create a tail (non-Maxwellian component) in beam ion velocity distribution function. It is well known that for suprathermal ions, the Nuclear Elastic Scattering (NES) by thermal ions also contributes to the slowing-down process. According to the recent scaling up of the fusion experimental devices, the use of beam energy more than 1 MeV is considered. In this case, the NES effects on the slowing down (energy transfer) process of injected beam particles becomes important to understand device performance during plasma heating operations. The purpose of this paper is to quantitatively estimate the NES effect on the neutral-beam-injection (NBI) plasma heating.

The NES is a non-Coulombic, large-energy-transfer (LET) scattering process. Devany & Stein[1] first pointed out the necessity of taking account the contribution of the nuclear forces to ion-ion scattering. In order to investigate the NES effect, several formulations to describe the discrete nature of LET scattering have been developed[2-4]. Using continuous[5] and multi-group[6,7] slowing-down models, energy-transfer processes of fusion-produced ions in DD and D-T He plasmas were investigated. In DT plasmas, a knock-on tail formation in deuteron (triton) distribution functions owing to the NES of α-particle and its effect on the emitted-neutron spectrum were examined by Fisher[8] and Ballabio[9]. Recently we have derived the energy loss rate of high-energy ions due to the NES, including the LET knocking-up of background ions from thermal to higher energy range[10], and using the derived expression we have estimated the NES effect on the fraction of beam energy deposited to ions in NBI heating operations. Since the NES is a discrete energy transfer process, however, the NES effect should be more accurately investigated considering the shape of the velocity distribution function of both energetic and background ions. On the basis of our previously-developed Boltzmann-Fokker-Planck (BFP) model[11,12], we made more accurate estimation[13]. Owing to the NES, thermal ions is knocked up to the high-energy range, and the high-energy tail, i.e. non-Maxwellian component, is formed in the fuel-ion distribution functions. In this case the T(d,n)⁴He reaction rate coefficient may be enhanced from the values when both D and T are Maxwellian. In this paper we further estimate the NES effect on the T(d,n)⁴He fusion reaction rate coefficient in NBI plasma heating operations, by simultaneously solving the Boltzmann-Fokker-Planck (BFP) equations for deuteron, triton, α-particle and beam ion in DT thermonuclear plasmas.

II. ANALYSIS MODEL

A. BFP Equation

To facilitate the analysis we only consider the nuclear elastic collision between beam ion and background fuel ions (D and T) as well as the collision between α-particle and background fuel ions. We simultaneously solve the following BFP equations for D, T, beam ion and α-particle:

\[
\left( \frac{\partial f_a}{\partial t_a} \right) = \sum \left( \frac{\partial f_a}{\partial v_a} \right)_{\text{NES}} + \frac{1}{v_a^2} \frac{\partial}{\partial v_a} \int f_a(v) \frac{v \cdot f_a(v)}{2E_a(v)} + S_a(v) \left( \frac{\partial f_a}{\partial v_a} \right)_{\text{R}} - L_a(v) = 0 \tag{1}
\]

where \( f_a \) is the velocity distribution function of species \( a \) (\( a=D,T,a,\text{beam-ion} \)). The first term in Eq.(1) represents the effect of the Coulombic collisions with ions (D,T,a,beam-ion) and electron. The second term accounts for the nuclear elastic collisions:

\[
\left( \frac{\partial f_a}{\partial t_a} \right)_{\text{NES}} = \frac{2\pi}{v_a^2} \int_{v_a}^{\infty} f_a(v') \left( \frac{v_a}{v} \right) P(v' \rightarrow v) \int_{v'}^{\infty} v''^2 \sigma_{\text{NES}}(v') dv' dv_v \tag{2}
\]

where \( v' = |v_a - v'| \) and \( v = |v_a + v'| \), subscript \( i \) represents background ion species, i.e. D, T for beam-ion and α-particle (beam-ion, α-particle for D and T). We have introduced the
probability distribution function $P$ (i.e., probability that the injected beam ion which has the speed $v'$ is scattered into the speed region $v_s$ owing to the NES with background ion, i.e. deuteron or triton, which has speed $v_s$). If we assume that the NES is isotropic in the CM system, then

$$P(v' \rightarrow v_s | v_s) = \left\{ \begin{array}{ll} \frac{2v_s}{v_{\max} - v_{\min}} & \text{for } v_{\min} \leq v_s \leq v_{\max}, \\ 0 & \text{otherwise} \end{array} \right. , \tag{3}$$

where

$$v_{\max(\min)} = \frac{m_s v_s^2 + m_i v_i^2}{m_s + m_i} \left( \frac{m_s}{m_s + m_i} \right)^{-\frac{1}{2}} \right). \tag{4}$$

The third term in Eq.(1) represents the diffusion in velocity space due to thermal conduction. The typical energy-loss time due to thermal conduction may be written as $\tau_e(v) = C_e \tau_c \text{Max} \{1, v/v_{ns}\}^{-\gamma}$. Here the coefficient $C_e$ is determined so that the velocity-integrated energy loss rate becomes $(3/2)nT/\tau_c$. By adjusting the parameter $\gamma$, we can simulate various loss mechanisms due to thermal conduction. For beam ion and $\alpha$-particle, particle source and loss terms can be written as

$$S_{\text{beam}}(v) - L_{\text{beam}}(v) = \frac{S_{\text{NBI}}(v)}{4\pi v} \delta(v - v_{\text{NBI}}) - \frac{f_{\text{beam}}(v)}{\tau_e(v)} , \tag{5}$$

where $S_{\text{NBI}}(v)$ is NBI rate ($\alpha$-particle generation rate) per unit volume and $v_{\text{NBI}}$ is speed corresponding to injected beam energy $E_{\text{NBI}}$ ($E_{\gamma}$: $\alpha$-particle birth energy). We express the injection rate by using the beam energy and dimensionless factor $Q_{\text{NBI}}$, i.e. $S_{\text{NBI}} = P_{\text{NBI}}/Q_{\text{NBI}} E_{\text{NBI}}$. Here $P_{\text{NBI}}$ represents the $\alpha$-heating power produced by $T(d,n)^4\text{He}$ reaction when $T = 0.1\text{keV}$ and $n_p = n = 2 \times 10^{19} \text{m}^{-3}$, i.e. $P_{\text{NBI}} = n_p \gamma_{\alpha} V_{\alpha} E_{\alpha} (E_{\alpha} = 3.52\text{MeV})$. For beam ion the particle loss due to fusion reaction is neglected. For bulk deuteron and triton, particle source and loss terms can be written as

$$S_{\text{D}}(v) - L_{\text{D}}(v) = \frac{S_{\text{NBI}}(v)}{4\pi v} \delta(v - v_{\text{D}}) - \frac{f_{\text{D}}(v)}{\tau_e(v)}, \tag{6}$$

where

$$S_{\text{D}}(v) = \frac{2\pi}{v_{\text{D}}} \int dv_{\text{D}} v_{\text{D}} Y_{\text{D}}(v_{\text{D}}, v) \left[ \bar{\nu}_{\text{D}} \right] \left[ \sigma_{\text{D}}(v_{\text{D}}, v) \right] \right]. \tag{7}$$

The fueling rate ($S_{\text{D}}$) is estimated so that the deuteron (triton) density is kept constant;

$$S_{\text{D}}(v) = \frac{n_{\text{D}}(v)}{\tau_e} + n_{\text{D}} \gamma_{\alpha} V_{\alpha} \left(\frac{E_{\alpha}}{n_{\text{D}} \gamma_{\alpha} V_{\alpha}}\right). \tag{8}$$

The confinement time of beam ion may be written as $\tau_e(v) = C_e \tau_c \text{Max} \{1, v/v_{ns}\}^{-\gamma}$. The coefficient $C_e$ is determined so that the velocity-integrated particle loss rate becomes $n/\tau_e$. In present calculations we assume that the loss of energetic ions is smaller compared with that of thermal ones, thus large $\gamma$ value, e.g. $\gamma=6$, is chosen. In this assumption, the influence of the $\gamma$ values on the fusion reactivity is small. Considering both energy loss mechanisms due to thermal conduction and particle transport, the global energy confinement time is defined as $1/\tau_e = 1/\tau_c + 1/\tau_p$. B. Fusion Reactivity

From the obtained deuteron and triton distribution functions, we can evaluate the $T(d,n)^4\text{He}$ fusion reactivity as

$$\sigma_{\text{D}}(v) = \frac{8\pi^2}{v_{\text{D}}} \left[ v_{\text{D}} Y_{\text{D}}(v_{\text{D}}, v) \right] \left( \sigma_{\text{D}}(v_{\text{D}}, v) \right). \tag{9}$$

To estimate the degree of the reaction enhancement due to tail (non-Maxwellian component) formation by NES, we compare the obtained fusion reaction rate coefficient with the one when both deuteron and triton distributions are assumed to be Maxwellian. For this purpose, we must identify the temperature of bulk (Maxwellian) component in deuteron and triton distribution functions. We determine the bulk temperature $T_{\text{bulk}}$ by comparing the bulk component of the obtained distribution function with Maxwellian by mean of the
least squares. To quantitatively estimate the reactivity enhancement, we introduce the decrement parameter:

$$\eta = \left( \frac{\langle \sigma \rangle_{\text{Maxwell}}}{\langle \sigma \rangle_{\text{Maxwell}}^{\text{D}}_{\text{T}}} \right) \times 100\% , \quad \text{(10)}$$

where, $\langle \sigma \rangle_{\text{Maxwell}}$ is the T(d,n)$^4$He fusion reaction rate coefficient when both deuteron and triton distribution functions are Maxwellian of temperature $T_{\text{bail}}$. In this paper the NES cross-sections are taken from the work of Perkins and Cullen[14], and the T(d,n)$^4$He fusion cross sections are taken from the work of Duane[15].

![Deuteron distribution function for several beam-injection energies. The dotted lines show Maxwellians.](image)

**III. RESULTS AND DISCUSSION**

We consider the DT plasma in which mono-energetic deuterium beam is injected. In Fig.1 we first show the steady-state (a) deuteron and (b) triton distribution functions when 1MeV mono-energetic beam is injected for background densities $n_e = 2n_p = 2 \times 10^{19} \text{ m}^{-3}$ and $8 \times 10^{19} \text{ m}^{-3}$. The solid lines indicate the present calculations, while the dotted lines denote Maxwellian temperature $T_{\text{bail}}$. Both distribution functions are normalized so that their densities become unity. In this calculation, the electron temperature $T_e = 20 \text{ keV}$, confinement times $\tau_e = (1/2)\tau_p = 3.0 \text{ sec}$ and $Q_{\text{stat}} = 1.0$ are assumed. We can find that suprathermal (non-Maxwellian) components appear in both deuteron and triton distribution functions owing to the recoils of thermal deuterons and tritons via NES by injected beam ions and $\alpha$-particles. The non-Maxwellian tail is created around several hundreds-keV energy range owing to the NES of bulk deuteron and triton by injected beam ion, while the tail in more than 1MeV energy range is formed due to the NES by $\alpha$-particles. The suprathermal component in deuteron distribution function is somewhat larger than that in the triton distribution function. This is because that the D-D NES cross-sections are larger than those of D-T ones. When background density is small, the slowing down of energetic ions is suppressed, and high-energy component in both deuteron and triton distribution functions grows large. In addition, in low-density plasmas the bulk temperature does not so easily increase, which further increases the fraction of tail to thermal component.

In Fig.2 we show the deuteron distribution function for several beam-injection energies. In this calculation, the electron temperature $T_e = 20 \text{ keV}$, confinement times $\tau_e = (1/2)\tau_p = 3.0 \text{ sec}$ and $Q_{\text{stat}} = 1.0$ are assumed. For large beam-injection energy, bulk ions are knocked up larger energy range, and thermal heating is weakened. As increasing beam-injection energy, tail becomes relatively large. In Fig.3 the enhancement parameter $\eta$ is presented as a function of beam-injection energies for several background densities. The calculation conditions are same as those in Fig.1 and Fig.2. According to the tail formation in both deuteron and triton distribution functions, the T(d,n)$^4$He fusion reaction rate coefficient between background deuteron and triton increases from the values when the NES is neglected. As was discussed in Fig.1 and Fig.2, for small background densities and high beam-injection energies, the fraction of the high-energy component in both deuteron and triton velocity distribution functions increases around the energy range where fusion cross-sections have a peak (~100keV deuterium energy). This is the reason why the enhancement parameter is large for small background densities and high beam-injection energies.

We next show the enhancement parameter $\eta$ as a function of electron temperatures for several background densities in Fig.4. The beam-injection energy $E_{\text{NBI}} = 1.0 \text{ MeV}$ and $Q_{\text{stat}} = 1.0$ are assumed. It is found that the enhancement in the fusion reaction rate coefficient becomes appreciable in 10-30keV temperature range. In low-temperature range the slowing down of energetic ions is intensified, thus the high-energy components in both deuteron and triton distribution functions becomes relatively small. On the other hand, at high-temperature range, relative velocity between beam ($\alpha$-particle) and background ions becomes small, and contribution of NES is reduced compared with Coulomb ones.
Finally, we show the enhancement parameter $\eta$ as a function of background densities in Fig. 5. The $T(d,n)^4\text{He}$ reaction rate coefficient when high-energy tails are created in both deuteron and triton distribution functions $\sigma_{v,T(d,n)^4\text{He}}$ and the coefficient when both deuteron and triton distribution function are assumed to be Maxwellian $\sigma_{v,T(d,n)^4\text{He}}^{\text{Maxwell}}$ are presented together with their enhancement factor $\eta$. The beam-injection energy $E_{\text{bif}}=1.0\text{MeV}$ and $Q_{\text{sat}}=1.0$ are assumed. As was discussed in Fig. 1, the degree of the reactivity enhancement is large for small density range. It is shown when $n_e \leq 5 \times 10^{19} \text{m}^{-3}$ the NES effect on the $T(d,n)^4\text{He}$ fusion reactivity between bulk deuteron and triton becomes appreciable.

![Figure 4. Reactivity enhancement factor $\eta$ as a function of electron temperatures for several electron densities.](image)

**IV. CONCLUSION**

We have studied the NES effect on the $T(d,n)^4\text{He}$ fusion reaction rate coefficient in NBI plasma heating operations for various plasma conditions. It has been shown that when $n_e = 2n_{ibf} = 2n_f \leq 5 \times 10^{19} \text{m}^{-3}$, the NES effect on fusion reactivity becomes appreciable. In this paper we only consider the $T(d,n)^4\text{He}$ reaction rate coefficient between background deuteron and triton. When the tail is created in the background triton distribution function, the fusion reaction between injected beam and background triton would also be influenced. Throughout the calculations $Q_{\text{sat}}=1.0$ has been assumed. The NES effect becomes smaller for large $Q_{\text{NBI}}$ values. If we consider the NBI heating during plasma startup operation for small $Q_{\text{NBI}}$ values, however, more significant NES effect on the fusion reaction rate coefficient may appear.

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**REFERENCES**